Sizing Linear and PWM Amplifiers Driving a Rotary Brushless Motor

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***Abstract*—This application note provides a design process for sizing linear and PWM three-phase amplifiers driving a rotary brushless motor (RBM). Design inputs consist of the motor torque constant, back-emf constant, number of poles, winding resistance and inductance, the load inertia, and the worst-case angular velocity and load torque profiles. The design outputs are the five key amplifier requirements: amplifier bus voltages, peak output current, continuous output current, peak output power, and continuous power dissipation. The *peak output power* and *continuous power dissipation* calculations apply only to linear amplifiers. Special attention is paid to trapezoidal angular velocity profiles and piecewise-constant load torque profiles, which are used in a companion spreadsheet [1]. Equations for power supply sizing and motor heating power are also provided.**

***Index Terms*—Amplifier Sizing, Linear Amplifier, PWM Amplifier, Rotary Brushless Motor, Varedan Document 4083-42-006 Revision C**

I. Introduction

This document provides background and design equations for sizing linear and PWM amplifiers driving a three-phase rotary brushless motor (RBM). The companion design spreadsheet [1] incorporates the design equations of Section IV below and provides an easy-to-use design tool that will meet the needs of most designers. The analyses contained here can provide valuable context and help the designer understand the limits of the design spreadsheet.

It is useful to think of amplifier-sizing as a design process that incorporates design inputs and design outputs. For an RBM system design, the following design inputs are used to size linear and PWM amplifiers. SI units are used throughout unless otherwise noted.

- Motor torque constant K_τ $(N-m/A_{rms})$
- Motor phase-to-phase back-emf constant K_e $(V_{\phi\phi\,peak}/(rad/sec))$
- Motor phase-to-phase resistance $R_{\phi\phi}$ (Ω)
- Motor phase-to-phase inductance $L_{\phi\phi}$ (H)
- Number of motor poles N
- Motor load inertia J $(kg-m^2)$
- Worst-case angular velocity profile $\omega(t)$ (rad/sec)
- Worst-case load torque profile $\tau(t)$ $(N-m)$

The design methodology described herein produces design outputs that consist of five key amplifier requirements below.

Peak output power and *continuous power dissipation* are design outputs that apply only to linear amplifier sizing as the specifications on current in PWM amplifiers are sufficient to determine power dissipation. These specifications that only apply to linear amplifiers are marked with an asterisk (*). The amplifier requirements are:

- Bipolar Amplifier Bus Voltage $\pm B$ for linear amplifiers, (Amplifier Bus Voltage $2B$ for PWM amplifiers)
- Peak Output Current I_{peak}
- Continuous Output Current I_{cont}
- Peak Output Power* P_{peak}
- Continuous Power Dissipation* P_{cont}

Once the five key amplifier requirements are determined, a linear or PWM amplifier is sized with specifications that meet or exceed the requirements.

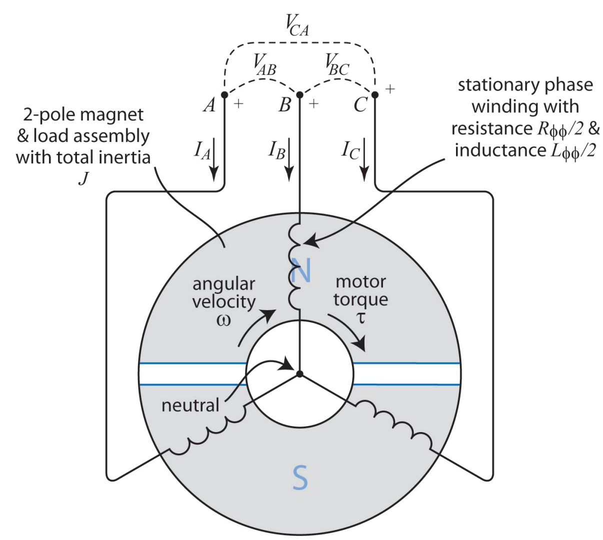

Fig. 1. Schematic diagram of a 2-pole 3-phase rotary brushless motor.

The remainder of this application note is organized as follows: In Section II, some background is presented and the design inputs are discussed and clearly defined. Algebraic equations that relate the design inputs to the design outputs are presented in Section III, and these equations are specialized to a trapezoidal angular velocity and piecewise-constant load torque profiles in Section IV. The equations for a trapezoidal profile are implemented in the spreadsheet [1]. Section V contains some simple design checks. A numerical sizing example is given in Section VI, and some auxiliary equations for power supply sizing and motor ohmic heating are provided in Section VII. Section VIII contains some conclusions.

II. Background & Design Inputs

The notation describing a three-phase RBM is introduced in this section together with definitions of the design inputs and outputs used for amplifier sizing. Consider first the simplified schematic of a rotating magnet RBM depicted in Fig. 1, which helps visualize the phase-to-phase voltages, the phase currents, and the load angular velocity.

The spacing between the phase windings is $120°$ and thus, together with the 2 poles, creates the symmetric electrical properties common to all three-phase RBMs.

Currents in phases A, B, and C produce torques on the moving magnets, and the motion of the magnets induce back-emf voltages in the phases. The motor torque and back-emf constants are defined in terms of the motor currents, voltages, torque, and angular velocity as follows.

A. Motor torque constant K_τ $(N - m/A_{rms})$

There are various definitions of motor torque constant which capture, in rough terms, the ratio *(motor torque)* $\div$ *(amount of motor current)*. Up to a choice of units, the "amount of motor torque" is clear. However, there are various ways to measure the currents in a motor. There are three sinusoidal phase currents and each current can be measured with an root-mean-square (rms) current measurement or a peak current measurement. A common measurement of current in the three phases is the rms current in a single phase *where it is understood that all phases are driven with a sinusoidal current of that same magnitude and having phases separated by* $2\pi/3\, radians\,(120°)$. To indicate this convention, the current unit in the torque-constant is labeled with rms as in $N - m/A_{rms}$ (that all phases are driven is implied).

To be clear, consider the typical sinusoidal phase-current waveforms I_A, I_B, and I_C applied to an RBM are depicted in Figure 2.

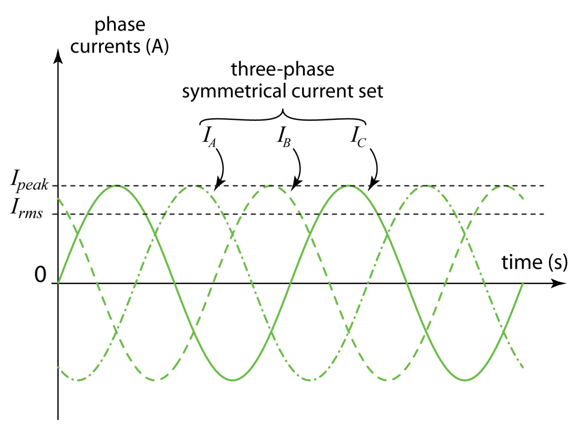

Fig. 2. Three-phase symmetrical current set used to drive an RBM. The figure depicts I_{rms}, which is used in the definition of K_τ.

The current waveforms have an obvious symmetry and are referred to as a three-phase symmetrical current set. The three sinusoidal currents collectively produce a constant torque τ, and their frequency is determined by the motor angular velocity ω and the commutation process. The rms current for each phase is the same and denoted by I_{rms}. The motor torque constant with an rms current measurement is defined by

$$K_\tau \equiv \tau/I_{rms}, \qquad (1)$$

where the symbol $\equiv$ is read "defined equal to." The units for the torque constant are "torque per amp rms" which is abbreviated $torque/A_{rms}$. In the following sections, the SI torque unit newton-meter is used and K_τ is expressed in $N - m/A_{rms}$.

Since the peak current is $\sqrt{2}$ times the rms current, a torque constant expressed as $torque/A_{peak}$ can be converted to $torque/A_{rms}$ by multiplying the first number by $\sqrt{2}$ as summarized in Table I. The rms unit is used in the next sections and in the companion spreadsheet [1]. Again, in either unit definition, it is assumed that the three phases are driven with a symmetrical current set.

When computing the motor torque τ for evaluation in Equation 1, both inertial and load torque terms are included. Let $\alpha(t) \equiv \frac{d\omega}{dt}$ denote the acceleration and then one obtains

$$F = K\alpha(t) + \tau_{load}. \qquad (2)$$

The mass includes any mass reflected through a linkage or gearbox mechanism as described below.

B. Motor phase-to-phase back-emf constant K_e $(V_{\phi\phi\,peak}/(rad/s))$

An RBM acts as a generator, and when the magnet array is rotated relative to the stationary phase windings, voltages are produced across the phase windings. That is, the motor creates an electromotive force (emf), which is considered backward by common sign conventions and is thus called the back emf. If the "generator" phase terminals are disconnected or otherwise unloaded, the sinusoidal phase-to-phase voltages have the same amplitude and frequency.

In the sections below, the back emf voltage is measured phase-to-phase and its peak value is used. The units are subscripted with $\phi\phi$ to indicate "phase-to-phase" and with $peak$ to indicate that the peak value of voltage is used. Figure 3 shows $V_{\phi\phi\,peak}$ graphically. SI units are used herein so that velocity is expressed in m/sec.

Note that the frequency of the phase-to-phase voltage sinusoids, as are their amplitudes, are proportional to the motor angular velocity.

Unit	Multiply By	To Get
$torque/A_{peak}$	$\sqrt{2}$	$torque/A_{rms}$

TABLE I
Torque Constant Conversion Factor

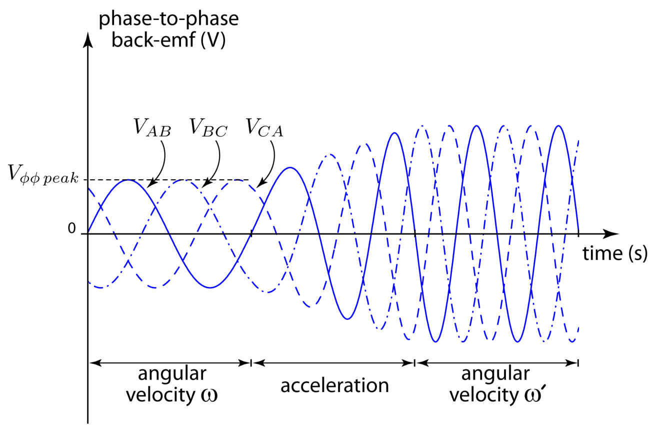

Fig. 3. Phase-to-phase voltage waveforms used in defining the back-emf constant K_e.

Since the back-emf voltage is proportional to angular velocity, and in reference to Figure 3, the back emf constant K_e is defined by

$$K_e \equiv V_{\phi\phi\, peak}/\omega. \tag{3}$$

The same K_e would be calculated at the higher velocity $omega'$ in Figure 3 with the proportionally higher peak voltage.

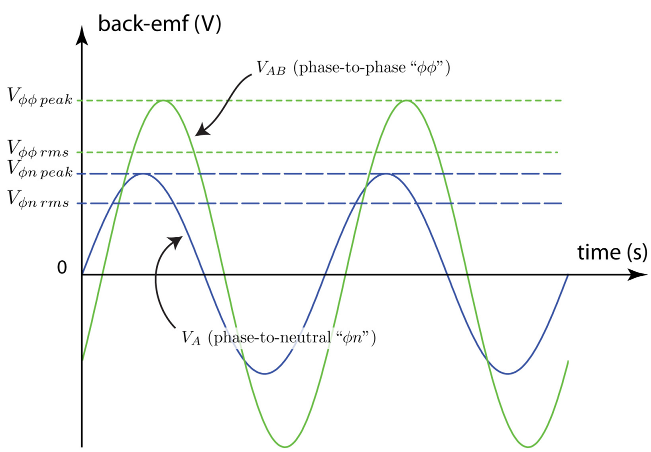

Fig. 4. Four ways to measure the back-emf voltage ($V_{\phi\phi\, peak}$, $V_{\phi\phi\, rms}$, $V_{\phi n\, peak}$, $V_{\phi n\, rms}$) – $V_{\phi\phi\, peak}$ is the measure used in this application note and in the spreadsheet [1].

Unfortunately, there are four natural definitions for the back-emf constant that quantify the ratio *(amount of back-emf voltage)* $\div$ *(motor speed)*. The definition based on phase-to-phase peak voltage is the one used in this application note and in [1], and there are three others. In defining the back-emf voltage, there is a choice of where to measure a voltage (phase-to-phase or phase-to-neutral) and a choice between rms voltage or peak voltage. Thus, there are *four* natural measurements (see Figure 4) not to mention various units used for angular velocity. Design engineers must be clear as to which measurement is used.

In reference to Figure 4, the peak voltage of a sinusoid is $\sqrt{2}$ times the rms voltage, and the amplitude of the phase-to-phase back-emf voltage $V_{\phi\phi}$ is $\sqrt{3}$ times the amplitude of the phase-to-neutral back-emf voltage $V_{\phi n}$. These ratios lead to the measurement conversions of Table II where the speed unit is arbitrary, but constant, throughout the table.

C. Phase-to-phase resistance $R_{\phi\phi}$ and inductance $L_{\phi\phi}$

Figure 1 shows the individual phase windings as having resistance $R_{\phi\phi}/2$ and inductance $L_{\phi\phi}/2$, such that the resistance and inductance measured across two phase terminals are $R_{\phi\phi}$ and $L_{\phi\phi}$ respectively.

D. Number of motor poles N

The total number of motor poles, both north and south, is N. Thus, there are $N/2$ north (and south) poles and the angular period of the magnet array in the motor is $4\pi/N$.

E. Motor load inertia J $(Kg - m^2)$

The rotational motor load inertia J is the total rotating inertia internal and external to the motor and includes any inertia reflected though a gearbox. Recall that the inertia reflected through a gearbox is the inertia on the output shaft times the speed ratio (output speed/input speed) squared. The inertia internal to the motor is usually listed in the motor data sheet.

F. Worst-case angular velocity and load torque profiles

A necessary design input for selecting an amplifier is one or more worst-case angular velocity and load torque profiles. The accelerations, peak velocities, and other features of these profiles inform the amplifier selection. In choosing worst-case angular velocity profiles, it is useful to understand how they affect the various design outputs. Further, such understanding can often guide the system design, and the design engineer may alter motion trajectories to reduce amplifier cost. A qualitative discussion of the angular velocity and load torque profile impacts on each of the design outputs is contained in subsections 1-5 below.

1) Peak Output Current: Short-time-constant thermal limits in connectors constrain the amplifier peak output current. Since large currents are needed to produce high torques, motion profiles with high peak accelerations and load torques will place demanding requirements on the peak

Unit	Multiply By	To Get
$V_{\phi\phi\, rms}/speed$	$\sqrt{2}$	$V_{\phi\phi\, peak}/speed$
$V_{\phi n\, peak}/speed$	$\sqrt{3}$	$V_{\phi\phi\, peak}/speed$
$V_{\phi n\, rms}/speed$	$\sqrt{6}$	$V_{\phi\phi\, peak}/speed$

TABLE II
BACK-EMF CONSTANT CONVERSION FACTORS

output current. Even a generally low-angular-velocity low-acceleration profile can be challenging in terms of peak output current if there are short bursts of high acceleration or load torque.

2) Continuous Output Current: Long time constants associated with conductor heating constrain the continuous output current. Trajectories that repeatedly accelerate and decelerate the load inertia, or require high load torques, can cause overheating of conductors by exceeding the high continuous output current specification of an amplifier.

3) Peak Output Power:* In linear amplifiers, the peak output power is that peak power experienced by a single output transistor, or set of paralleled output transistors when acting as a single device. Heavy braking at high speeds causes large voltage drops and high currents in the output transistors, leading to high power dissipation. Emergency stops from high speed and angular velocity trajectories with similar motions require high peak output power. This peak output power calculations provided herein do not apply to PWM amplifiers. High load torques with large output transistor voltage drops also cause high power dissipation in the output transistors.

4) Continuous Power Dissipation:* In linear amplifiers, trajectories that repeatedly impose high inertial or load torques cause high continuous power dissipation in the output transistors. Over time, the temperature of the heat sink rises and the junction temperature of the power transistors can exceed specifications. Such trajectories also challenge the continuous output current specification. The continuous power dissipation calculations provided herein do not apply to PWM amplifiers.

5) Bipolar Amplifier Bus Voltage for linear amplifiers, Amplifier Bus Voltage for PWM amplifiers: The bus-to-bus voltage in a linear or PWM amplifier must exceed the largest back-emf voltages, which are sinusoidal with zero mean. Thus, high-speed trajectories and/or motors with large back-emf constants can exceed the capabilities of the power supply/amplifier system. In linear amplifiers, the bus voltages are $+B\,Volts$ and $-B\,Volts$ and are referred to as the "bipolar amplifier bus voltage $\pm B$". In PWM amplifiers, the bus voltages are $0\,Volts$ and $2B\,Volts$ where the latter is normally referred to as the bus voltage and the bus at $0\,Volts$ is implied. The variable B is used in specifying both amplifier types in order to simplify notation. In either case, the bus-to-bus voltage is $2B$.

6) Trapezoidal Angular Velocity Profiles: A common trapezoidal angular velocity profile will be used in conjunction with a piecewise-constant load torque profile to estimate the five key amplifier requirements. Trapezoidal profiles can be constructed to produce any of the demanding motions described in subsections 1-5 above and approximate many of the motions that the designer may wish to generate. Figure 4 depicts an example of a trapezoidal profile with the timing and amplitude variables that precisely describe the motion. The analysis in Section IV refers to the profile of Figure 4.

The trapezoidal profile is assumed to be periodic with period T, which is 1.80 seconds for the profile in Figure 4.

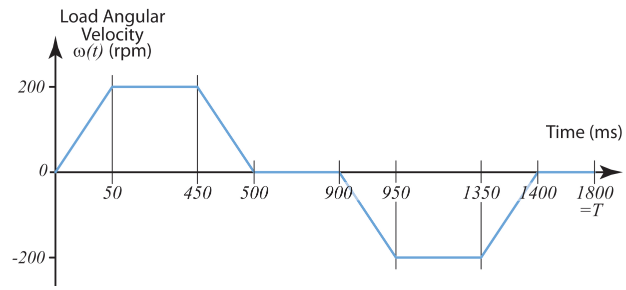

Fig. 5. Typical trapezoidal angular velocity profile.

7) Piecewise-Constant Load Torque Profiles: In addition to inertial torques required to accelerate the inertia J, the motor may also be used to apply, say, cutting torques in a machining operation. For the sake of amplifier sizing, the load torque profile $\tau_{load}(t)$ is assumed to be piecewise constant and have transition times at the corner times of the velocity profile. See Figure 5 for an example of a piecewise-constant torque profile. In referring to the constant torque levels in the profile, we define $\tau_{load}(t+)$ to be the torque immediately following time t. For example, in the profile in Figure 5, $\tau_{load}(200ms+) = 1.5\,N-m$, and $\tau_{load}(300ms+) = 0\,N-m$.

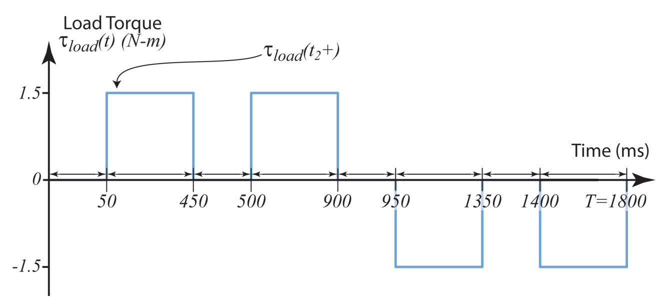

Fig. 6. Example of a worst-case piecewise-constant load torque profile – a design input. $\tau_{load}(t+)$ denotes the torque immediately following time t_k.

III. Design Outputs

In this section, the design outputs are expressed, via equations, in terms of the design inputs. These equations serve as design tools for sizing linear and PWM amplifiers.

A. Bipolar Amplifier Bus Voltage $\pm B$ for linear amplifiers, Amplifier Bus Voltage $2B$ for PWM amplifiers

The bipolar amplifier bus voltage $\pm B$ determines the limits of a linear amplifiers' output voltage range. The positive bus voltage is $+B$ or simply B, the negative bus voltage is $-B$, and the voltage between the buses is $2B$. For PWM amplifiers the bus voltages are 0 and $2B$ so

*Calculations marked with an asterisk (*) only apply to linear amplifiers that the voltage between the buses is also $2B$. The bus voltages must accommodate the phasor sum of the phase-to-phase back-emf voltage and the phase-to-phase voltage drops for the phase resistance and inductance. More precisely, let $\max_t x(t)$ denote the maximum value of the function $x(t)$ for the allowable values of t and $\alpha(t) \equiv \frac{dv}{dt}$ denote the acceleration. The bus voltages are calculated from the peak phase-to-neutral voltage, which is given by

$$V_{\phi n\, peak} = \max_t \left[\left(\frac{\sqrt{2}(J\alpha(t) + \tau_{load}(t))R_{\phi\phi}}{2K_\tau} + \frac{\omega(t)K_e}{\sqrt{3}} \right)^2 + \left(\frac{(J\alpha(t) + \tau_{load}(t))\omega(t)NL_{\phi\phi}}{2\sqrt{2}K_\tau} \right)^2 \right]^{1/2}. \quad (4)$$

Since K_τ is defined in terms of an rms current, a $\sqrt{2}$ appears in the expression above to obtain the needed peak current value. Similarly, K_e is a phase-to-phase back-emf constant, and a needed phase-to-neutral value is achieved by dividing by $\sqrt{3}$. Factors of 2 are used to convert the phase-to-phase $R_{\phi\phi}$ and $L_{\phi\phi}$ to phase-to-neutral values.

A margin of safety of, say, 20% is often included to account for variation in the line voltage and for other uncertainties. Thus, the bipolar amplifier bus voltage for a linear amplifier might be chosen to be

$$\pm B = \pm 1.2 V_{\phi n\, peak}. \quad (5)$$

The corresponding bus voltage for a PWM amplifier is

$$V_{bus} = 2B = 2.4 V_{\phi n\, peak}. \quad (6)$$

B. Peak Output Current I_{peak}

The peak output current is defined as the maximum value of any phase current over the duration of motion profile. The peak output current is given by

$$\begin{aligned} I_{peak} &\equiv \max_t \sqrt{2}|\tau(t)|/K_\tau \\ &= \max_t \sqrt{2}|J\alpha(t) + \tau_{load}(t)|/K_\tau \end{aligned} \quad (7)$$

where $|\cdot|$ denotes absolute value and the rightmost expression is in terms of the design inputs alone as desired.

C. Continuous Output Current I_{cont}

Long-time-constant thermal limits of conductors and other components affect the maximum continuous output current that an amplifier can supply. This specification is expressed as an rms value for one phase. Continuous operation above this threshold is expected to cause amplifier failures. For non-constant operation, the specification is interpreted as an rms current computed over a periodic angular velocity profile. That is, the “mean” in the root-mean-square current is taken over the period T of the motor motion:

$$I_{cont} = \left(\frac{1}{T} \int_0^T \left(\frac{J\alpha(t) + \tau_{load}(t)}{K_\tau} \right)^2 dt \right)^{1/2}. \quad (8)$$

D. Peak Output Power* P_{peak}

In linear amplifiers, the peak output power is defined as the maximum instantaneous power dissipated in any single amplifier output transistor, or set of paralleled transistors acting as a single device, over the entire motion profile.

Linear amplifier output transistors have a limit on peak power dissipation, and this limit is approached when there is a large voltage drop across an output transistor while a high current is passing through that transistor. Such a high-voltage high-current condition arises when the motor velocity is large in magnitude, positive or negative, and the motor aggressively brakes. In this situation, the large back-emf and the bus voltages add constructively across the output transistor, and large currents are required. However, the voltage across the output transistor is reduced by the phase-to-phase voltage drop across the phase-to-phase resistance $R_{\phi\phi}$. The phase-to-phase inductance $L_{\phi\phi}$ is neglected in the peak power calculation and, on occasion, this approximation should be reconsidered.

Express rms currents in terms of the torque τ and the torque constant K_τ, and express peak phase-to-phase back-emf voltages in terms of the angular velocity $\omega(t)$ and the back-emf constant K_e. Further, recall that $2B$ is the voltage across the two power supply buses. Then the power dissipated in the active transistor at a current crest is computed to be

$$\sqrt{2}B|\tau|/K_\tau - R_{\phi\phi}\tau^2/K_\tau^2 - \sqrt{2}\omega\tau K_e/(K_\tau\sqrt{3}). \quad (9)$$

The $\sqrt{2}$ and $\sqrt{3}$ appear for the same reasons noted above. Note that the third term in the equation above increases the power dissipation if $\omega(t)$ and $\tau(t)$ have opposite signs as expected. Replace $\tau(t)$ with $J\alpha(t) + \tau_{load}(t)$, and the expression for peak output power becomes, in terms of design inputs and the design output B,

$$\begin{aligned} P_{peak} = \max_t \Big[&\sqrt{2}B|J\alpha(t) + \tau_{load}(t)|/K_\tau \\ &- R_{\phi\phi}(J\alpha(t) + \tau_{load}(t))^2/K_\tau^2 \\ &- \sqrt{2}\omega(t)(J\alpha(t) + \tau_{load}(t))K_e/(K_\tau\sqrt{3}) \Big]. \end{aligned} \quad (10)$$

This peak output power calculation does not apply to PWM amplifiers.

1) Frequency Adjustments to Peak Output Power:* For short current pulses in linear amplifiers, the thermal mass of the transistor junction and surrounding silicon lowers the peak junction temperature relative to that for a DC current of the same magnitude. This beneficial frequency dependence is quantified in the Transient Thermal Impedance for the transistor. For the devices used in Varedan amplifiers, the thermal mass effects are insignificant at commutation frequencies of about $5/3$ Hz and below. However, the effects are significant at 10Hz and above and can be exploited to achieve higher peak output power. Varedan does exploit the thermal mass effects when computing the safe operating area (SOA) of its linear amplifiers in order to avoid unnecessary fault conditions. Further, the thermal mass effects can be used in sizing amplifiers. Using the commutation frequency $f = \omega N/(4\pi)$, the junction-to-heat-sink thermal impedance for the MOSFET is conservatively approximated by

$$R_{j-HS}(f) = 10^{\left(0.08657 \ln\left(\frac{500}{f}\right) - 1.021\right)} + 0.05 \frac{^\circ C}{W}, \quad (11)$$

for $f \geq 5/3$. For $f < 5/3$, the impedance is constant and given by $R_{j-HS}(f) = R_{j-HS}(5/3)$. Thus, $5/3$ Hz is a corner in the frequency response estimate. Define the normalized response function as

$$n(f) \equiv R_{j-HS}(f)/R_{j-HS}(5/3), \quad (12)$$

which is equal to 1 for frequencies below $5/3$ Hz and is less than 1 for frequencies above $5/3$ Hz. The normalized response function $n(f)$ is used as a scale-factor to multiply the expression for P_{peak} in Equation 10 when comparing application requirements to the DC peak power specified for a given rotary amplifier. Interestingly, this normalization can be used for all MOSFETs in the power range of Varedan's products. This is likely due to the thermal properties of silicon and similarities in the device geometries. This factor only applies to linear amplifiers and is not included in the standard spreadsheet [1] to add conservatism to routine sizing calculations.

E. Continuous Power Dissipation P_{cont}*

In linear amplifiers, continuous power dissipation in the output transistors has the potential to overheat the heat sink system and the transistor junctions. The average, over one commutation cycle, of the dissipation in the two output transistors of a single phase is given by

$$\frac{2\sqrt{2}}{\pi K_\tau}|\tau|B - \frac{R_{\phi\phi}\tau^2}{2K_\tau^2} - \frac{\omega \tau K_e}{\sqrt{6}K_\tau}. \quad (13)$$

Since the thermal time constant of the heat sink is long, the continuous power dissipation for a periodic angular velocity profile $\omega(t)$ is taken to be the average over the period T of the power dissipated in the three output stages. Using $\tau = J\alpha + \tau_{load}$, one obtains

$$\begin{aligned} P_{cont} = \frac{3}{T}\int_0^T \Bigg(& \frac{2\sqrt{2}}{\pi K_\tau}\,|J\alpha(t) + \tau_{load}(t)|B \\ & - \frac{R_{\phi\phi}(J\alpha(t) + \tau_{load}(t))^2}{2K_\tau^2} \\ & - \frac{\omega(t)(J\alpha(t) + \tau_{load}(t))K_e}{\sqrt{6}K_\tau}\Bigg)\, dt. \end{aligned} \quad (14)$$

The continuous power dissipation calculation only applies to linear amplifiers.

IV. Evaluating the Design Outputs for Trapezoidal Angular Velocity and Piecewise-Constant Load Torque Profiles

Note that a trapezoidal angular velocity profile is equivalent to piecewise-constant angular acceleration and torque profiles. In this case, the design equations simplify and a systematic design process can be implemented in the companion spreadsheet [1]. Refer to the periodic trapezoidal angular velocity profile in Figure 4 and note that the profile can be specified by the nine corners that have (time, angular velocity) coordinates (0s, 0rpm), (50ms, 200rpm), (450ms, 200rpm),. . . , (1.8s, 0rpm), or, for short and with a conversion to units of rad/sec, $(t_k, \omega(t_k))$; $k = 1, 2, \ldots, 9$. Similarly for $(t_k, \tau_{load}(t_k+))$; $k = 1, 2, \ldots, 9$, where $\tau_{load}(t_k+)$ denotes the load torque just after the time t_k. Due to discontinuities at t_k, the value of the load torque at t_k is not well defined. Again, the trajectory is assumed to be periodic with, in the case of Figures 4 and 5, a period of $T = 1.8s$.

The first observation to make is that the design outputs can be calculated using $\omega(t)$ and $\tau_{load}(t)$ near the corner points of the angular velocity profile. Specifically, the design outputs are determined by the velocities $\omega(t_k)$, and the acceleration $\alpha(t)$ just before and just after each corner, which are denoted $\alpha(t_k-)$ and $\alpha(t_k+)$ respectively. The torques $\tau_{load}(t_k+)$ and $\tau_{load}(t_k-)$ also enter the calculations and the inertial torques due to acceleration and load torques are combined into a single torque. Since the trajectory is periodic, the acceleration just after the 9th corner is the same as that just after the 1st corner. That is $\alpha(t_9+) = \alpha(t_1+)$ - similarly $F(t_9+) = F(t_1+)$. The acceleration and load torques are often discontinuous right at the corners and not defined there – although inductance in the motor windings and other sources of filtering will round the corners in practice.

When restricted to trapezoidal angular velocity and piecewise constant load torque profiles, the five design equations above are simplified as follows.

A. Bus Voltages for Trapezoidal Angular Velocity and Piecewise-Constant Load Torque Profiles

The peak output voltage amongst all the profile corners is obtained by maximizing over all values of the corner velocities $\omega(t)$ and the accelerations and load torques just prior to and just after the corners. Thus, the calculation involves 16 different evaluations of the expression in square brackets below.

$$V_{\phi n\,peak} = \max_{k,\,\pm}\left[\left(\frac{\sqrt{2}(J\alpha(t_k\pm)+\tau_{load}(t_k\pm))R_{\phi\phi}}{2K_\tau}+\frac{\omega(t_k)K_e}{\sqrt{3}}\right)^2 + \left(\frac{(J\alpha(t_k\pm)+\tau_{load}(t_k\pm))\omega(t_k)NL_{\phi\phi}}{2\sqrt{2}K_\tau}\right)^2\right]^{1/2}. \quad (15)$$

In the maximization over $\pm$, either "+" or "-" is used throughout Equation 15 and in similar maximizations in this note.

Applying a recommended margin of 20% yields

$$\pm B = \pm 1.2 V_{\phi n\,peak} \quad (16)$$

for the bipolar amplifier bus voltage for a linear amplifier. The bus voltage for a PWM amplifier is

$$V_{bus} = 2B = 2.4 V_{\phi n\,peak}. \quad (17)$$

B. Peak Output Current for Trapezoidal Angular Velocity and Piecewise-Constant Load Torque Profiles

The relationship for the peak output current becomes

$$I_{peak} = \max_{k,\,\pm}\sqrt{2}|J\alpha(t_k\pm)+\tau_{load}(t_k\pm)|/K_\tau. \quad (18)$$

C. Continuous Output Current for Trapezoidal Angular Velocity and Piecewise-Constant Load Torque Profiles

The acceleration is constant between corner times in a trapezoidal angular velocity profile. Thus, the integral expression for the continuous output current becomes the summation

$$I_{cont} = \left(\frac{1}{T}\sum_{k=1}^{8}\left(\frac{J\alpha(t_k+)+\tau_{load}(t_k+)}{K_\tau}\right)^2(t_{k+1}-t_k)\right)^{1/2}, \quad (19)$$

where $T = t_9$ is the period of the angular velocity profile.

D. Peak Output Power for Trapezoidal Angular Velocity and Piecewise-Constant Load Torque Profiles*

The peak output power occurs just before or just after one of the corners.

$$P_{peak} = \max_{k,\,\pm} n\left(\frac{\omega(t_k)}{p}\right)\times \left[\frac{\sqrt{2}B|J\alpha(t_k\pm)+\tau_{load}(t_k\pm)|}{K_\tau} - \frac{R_{\phi\phi}(J\alpha(t_k\pm)+\tau_{load}(t_k\pm))^2}{K_\tau^2} - \frac{\sqrt{2}\omega(t_k)(J\alpha(t_k\pm)+\tau_{load}(t_k\pm))K_e}{K_\tau\sqrt{3}}\right], \quad (20)$$

where the normalized response function (not included in the spreadsheet [1]) is used to incorporate the benefit of the thermal mass of the transistor junctions. The peak output power calculation only applies to linear amplifiers.

E. Continuous Power Dissipation for Trapezoidal Angular Velocity and Piecewise-Constant Load Torque Profiles*

Integrating the expression for continuous power dissipation for a trapezoidal angular velocity profile yields

$$P_{cont} = \frac{3}{T}\sum_{k=1}^{8}\left(\frac{2\sqrt{2}}{\pi K_\tau}|J\alpha(t_k+)+\tau_{load}(t_k+)|B - \frac{R_{\phi\phi}(J\alpha(t_k+)+\tau_{load}(t_k+))^2}{2K_\tau^2} - \frac{(\omega(t_k)+\omega(t_{k+1}))(J\alpha(t_k+)+\tau_{load}(t_k+))K_e}{2\sqrt{6}K_\tau}\right)\Delta t_k, \quad (21)$$

where $\Delta t_k = (t_{k+1}-t_k)$. The continuous power dissipation calculation only applies to linear amplifiers.

V. Design Checks

In implementing the design equations above, there are various ways that errors may occur. In addition to the usual transcription and algebraic errors, confusion may arise from the various definitions of torque and back-emf constants, as well as from the unit systems used. There are a couple simple checks that are worth performing.

A. $\frac{K_\tau}{K_e} = \sqrt{\frac{3}{2}} \approx 1.22$

For the definitions and SI units used herein the above relationship between the torque and back-emf constants holds for ideal three-phase motors. Since magnetic materials exhibit nonlinearity and the back-emf waveforms can deviate from the ideal sinusoids because of motor geometry, variations of up to a few percent might be observed in some motor data sheets. However, if there is confusion in units, or confusion in the use of rms vs. peak or phase-to-phase vs. phase-to-neutral in parameter definitions, checking the ratio $\frac{K_\tau}{K_e}$ will reveal the error.

B. $\frac{L_{\phi\phi}}{R_{\phi\phi}} = \tau_e$

Motor data sheets are usually redundant in quoting resistance, inductance, and motor electrical time constant τ_e. The relationship above holds up to round-off and measurement error. If millihenries are used instead of henries and milliseconds are used instead of seconds in the calculations, errors will escape detection with this check.

C. $\dfrac{L_{\phi\phi}}{R_{\phi\phi}} \ll \min\limits_{k=1,\ldots,8}(t_{k+1} - t_k)$

For good tracking, the electrical time constant of the motor should be much shorter than any of the time periods defining the trapezoidal angular velocity profile. This is rarely a problem, but it is easy to check.

VI. Sizing Example

To illustrate the use of the design equations, consider the amplifier sizing to address the following design inputs:

- Motor torque constant $K_\tau = 1.23\,N - m/A_{rms}$
- Motor phase-to-phase back-emf constant $K_e = 1.0\,V_{\phi\phi\,peak}/(rad/s)$
- Motor phase-to-phase resistance $R_{\phi\phi} = 1.5\,\Omega$
- Motor phase-to-phase inductance $L_{\phi\phi} = 23\,mH$
- Number of motor poles $N = 20$
- Motor load inertia $J = 0.05\,kg - m^2$
- Worst-case angular velocity profile $\omega(t)$ given by Figure 4.
- Load torque profile equal to zero: $\tau_{load}(t) = 0$.

By converting the units in Figure 4 to SI units, one sees that the angular velocity ranges over $\pm 200\,rpm \times \frac{2\pi}{60} = \pm 20.9\,rad/s$. The angular acceleration ranges over $\pm 419\,rad/s^2$.

A. Example: Amplifier Bus Voltages

The peak output voltage occurs when acceleration and angular velocity are both high. This peak occurs in the profile of Figure 4 just prior to corner two and just prior to corner six when the peak voltage reaches the same value. Thus, using the formula for peak phase-to-neutral voltage $V_{\phi n\,peak}$ and setting the load torque to zero, one has:

$$V_{\phi n\,peak} = \left[\left(\frac{\sqrt{2} J \alpha(t_2-) R_{\phi\phi}}{2K_\tau} + \frac{\omega(t_2) K_e}{\sqrt{3}} \right)^2 + \left(\frac{J\alpha(t_2-)\omega(t_2-) N L_{\phi\phi}}{2\sqrt{2} K_\tau} \right)^2 \right]^{1/2}. \quad (22)$$

Substituting the numerical values yields

$$V_{\phi n\,peak} = \left[\left(\sqrt{2} \cdot 0.05 \cdot 419 \cdot 1.5/(2 \cdot 1.23) + 20.9 \cdot 1.0/\sqrt{3} \right)^2 + \left(0.05 \cdot 419 \cdot 20.9 \cdot 20 \cdot 0.023/(2 \cdot \sqrt{2} \cdot 1.23) \right)^2 \right]^{1/2} = 65.4\,V. \quad (23)$$

Applying a recommended margin of 20%, the bipolar amplifier bus voltage for a linear amplifier is

$$\pm B = \pm 78.4\,V. \quad (24)$$

For a PWM amplifier, the bus voltage is

$$V_{bus} = 2B = 156.8\,V. \quad (25)$$

.

B. Example: Peak Output Current

The peak current occurs at peak acceleration, which is equal to $419\,m/s$ for the profile of Figure 4. This current peak occurs during all of the angular velocity ramps. In particular, the peak occurs just following corner one, such that

$$I_{peak} = \frac{\sqrt{2} J |\alpha(t_1+)|}{K_\tau} = \sqrt{2} \cdot 0.05 \cdot \frac{419}{1.23} = 24.1\,A \quad (26)$$

C. Example: Continuous Output Current

The rms current is either $17.8A/\sqrt{2}$ during the four $50\,ms$ ($0.2\,s$ total) angular velocity ramps or $0A$ during periods of constant angular velocity. The continuous current is therefore

$$I_{cont} = \left(\frac{1}{T} \sum_{k=1}^{8} \left(\frac{J\alpha(t_k+)}{K_\tau} \right)^2 (t_{k+1} - t_k) \right)^{1/2} = \left(\frac{1}{1.8} \left(\frac{0.05 \cdot 419}{1.23} \right)^2 (0.2) \right)^{1/2} = 5.68\,A \quad (27)$$

D. Example: Peak Output Power*

For linear amplifiers, the peak output power typically occurs just after corners 3 and 7 in trajectories such as that in Figure 4. However, due to the frequency adjustment, it is not clear where the maximum dissipation and the power should be evaluated just following all corners. In this case, it is known that either corner 1 (equivalently corner 5) or corner 3 (equivalently corner 7) attains the maximum power. First, evaluate the power just following corner 3. The commutation frequency at corner 3 is

$$f = \frac{\omega(t_3) N}{4\pi} = \frac{20.9 \cdot 20}{4\pi} = 33.3\,Hz. \quad (28)$$

Using this value of f, which is greater than the corner frequency of $5/3$, one obtains

$$R_{j-HS}(33.3) = 10^{\left(0.08657 \ln\left(\frac{500}{33.3}\right) - 1.021\right)} + 0.05 = 0.170°C/W, \quad (29)$$

and

$$n(f) \equiv \frac{R_{j-HS}(33.3)}{R_{j-HS}(5/3)} = 0.827. \quad (30)$$

The peak frequency-adjusted power (not computed in the spreadsheet [1]) is then computed as

$$P_{peak} = n\left(\frac{\omega(t_3) N}{4\pi} \right) \left[\frac{\sqrt{2} B J |\alpha(t_3+)|}{K_\tau} - \frac{R_{\phi\phi} J^2 \alpha(t_3+)^2}{K_\tau^2} - \frac{\sqrt{2} J \omega(t_3) \alpha(t_3+) K_e}{K_\tau \sqrt{3}} \right] \quad (31)$$

$$= 0.827 \cdot \left[\frac{\sqrt{2} \cdot 78.4 \cdot 0.05 \cdot 419}{1.23} - \frac{1.5 \cdot 0.05^2 \cdot (-419)^2}{1.23^2} - \frac{\sqrt{2} \cdot 0.05 \cdot 20.9 \cdot (-419) \cdot 1.0}{(1.23 \cdot \sqrt{3})} \right] = 1443\,W \tag{32}$$

A similar calculation for corner 1 yields a slightly larger 1454 Watts and one sees that the peak power is attained at corners 1 and 5. Therefore,

$$P_{peak} = 1454\,W. \tag{33}$$

This peak output power calculation does not apply to PWM amplifiers.

E. Example: Continuous Power Dissipation*

Finally, for linear amplifiers and zero load torque, the continuous power dissipation formula is

$$P_{cont} = \frac{3}{T} \sum_{k=1}^{8} \left(\frac{2J\sqrt{2}}{\pi K_\tau} |\alpha(t_k+)| B - \frac{R_{\phi\phi} J^2 \alpha(t_k+)^2}{2K_\tau^2} - \frac{J(\omega(t_k) + \omega(t_{k+1}))\alpha(t_k+) K_e}{2\sqrt{6} K_\tau} \right) \Delta t_k, \tag{34}$$

where $\Delta t_k = t_{k+1} - t_k$. Equation (34) can be simplified by observing that power is dissipated during acceleration on the angular velocity ramps only. Further, there are two kinds of ramps. There is one case in which acceleration and angular velocity have the same sign and another case in which they have the opposite signs. Thus, the continuous power dissipation can be calculated by computing just these two cases and doubling the result (the first factor of 2 in the equation below). The two cases are distinguished by different signs prior to the terms in square braces below (these terms cancel):

$$P_{cont} = 2 \cdot \frac{3}{1.8} \left\{ \left(\frac{2 \cdot 0.05 \cdot \sqrt{2}}{\pi \cdot 1.23} \cdot 419 \cdot 78.4 - \frac{1.5 \cdot 0.05^2 \cdot 419^2}{2 \cdot 1.23^2} - \left[\frac{\sqrt{2} \cdot (0.05/2 + 0/2) \cdot 419 \cdot 1.0}{2\sqrt{3} \cdot 1.23} \right] \right) (0.050) + \left(\frac{2 \cdot 0.05 \cdot \sqrt{2}}{\pi \cdot 1.23} \cdot 419 \cdot 51.2 - \frac{1.5 \cdot 0.05^2 \cdot 419^2}{2 \cdot 1.23^2} + \left[\frac{\sqrt{2} \cdot (0.05/2 + 0/2) \cdot 419 \cdot 1.0}{2\sqrt{3} \cdot 1.23} \right] \right) (0.050) \right\} = 328\,W \tag{35}$$

This continuous power dissipation calculation does not apply to PWM amplifiers.

VII. Power Supply Sizing & Motor Ohmic Heating

The primary objective of this application note is to provide requirements for choosing an amplifier. It is possible, in addition, to compute power supply requirements and motor heat dissipation requirements, since these additional requirements can be determined from the same design inputs used in the amplifier sizing equations.

A. Power Supply Sizing

For a linear amplifier, the averaged absolute value of a phase current over one commutation cycle is the peak current divided by $\pi/2$. The worst-case power over one commutation cycle into each phase amplifier is then $2BI_{peak}/\pi$, and for all three phases one has $6BI_{peak}/\pi$. Since the power supply is bipolar for linear amplifiers, the power per bus is half the total or

$$P_{bus\,linear} = 3BI_{peak}/\pi. \tag{36}$$

The current per bus is

$$I_{bus\,linear} = 3I_{peak}/\pi. \tag{37}$$

For a PWM amplifier, the power delivered on the single bus of voltage $2B$ is the same as the power delivered by the two buses of voltage $\pm B$ to a linear amplifier. Thus,

$$P_{bus\,PWM} = 6BI_{peak}/\pi. \tag{38}$$

Dividing by the bus voltage $2B$ for a PWM amplifier, one finds that the current requirement for a PWM amplifier is the same as that for a linear amplifier:

$$I_{bus\,PWM} = 3I_{peak}/\pi. \tag{39}$$

B. Motor Ohmic Heating $P_{motor-heat}$

The power input to the motor is converted to power output in the form of either mechanical power in the motor shaft or heat that must be transferred from the motor to the ambient environment. The dominant source of heat generated in the motor is the I^2R losses in the three windings, which are given by

$$P_{motor-heat} = \frac{3}{2} I_{cont}^2 R_{\phi\phi}. \tag{40}$$

The division by 2 converts the phase-to-phase resistance $R_{\phi\phi}$ to the phase-to-neutral value. There are other sources of heat generation in a motor associated with eddy-currents, windage, friction, and hysteresis losses in motor. While these sources are generally negligible in a conservative thermal design, they can be included by adding power losses associated with viscous and Coulomb friction losses listed in the data sheet. Eddy-currents contribute to viscous losses, and hysteresis is usually lumped with friction measurements.

VIII. Conclusion

The equations of Section IV provide a simple means to size amplifiers for driving RBMs. The *peak output power* and *continuous power dissipation* calculations only apply to the sizing of linear amplifiers and such calculations are marked with an asterisk (*) throughout. Specifications on current are sufficient to estimate power dissipation in PWM amplifiers. In many applications, S-curves rather than lines define accelerations. The rounding of the corners by using such S-curves generally reduces the demand on linear amplifiers. If one is attempting a close fit of the motion requirement to the amplifier specification, a more complete simulation (e.g. in Matlab) will provide more information. In the case of S-curves, the equations of Section III can be used, and inductance effects can be added. Finally, care should be taken with long-period motions and aperiodic motions, as an assumption was made that the thermal time-constant (~ 60 seconds) of the heat sink systems is long enough that the heat sink temperature is determined by P_{cont}.

Questions and feedback on this application note are most welcome and can be directed to sales@varedan.com.

References

[1] "Spreadsheet: Sizing Linear and PWM Amplifiers Driving a Rotary Brushless Motor," Varedan Technologies Document 4083-42-002.

[2] Hurley Gill, "Servomotor Parameters and their Proper Conversions for Servo Drive Utilization and Comparison," Kollmorgen Inc.